EIJEST

The Egyptian Int. J. of Eng. Sci. and Technology,
Vol.14, No. 1 (Jan. 2011)

PREDICTION OF PUNCHING SHEAR RESISTANCE OF PRE-STRESSED CONCRETE FLAT SLABS*

Ashraf. M. EL-shihy , Hamdy. A. K. Shehabeldin, Osman. E. A. Shaalan and Ragab. S. Mahmoud**

Structural Eng.Dept. Faculty of Engineering. Zagazig University, Egypt

ABSTRACT:

Punching shear of pre-stressed concrete flat slab is an undesirable mode of failure that occurs without warning and can lead to progressive collapse of large areas of the slab or even complete structures. Modeling the punching shear strength of pre-stressed concrete slabs is conceptually difficult as the tendons can be bonded or unbonded, the tendon passing through the columns or not, the stress in tendons at the punching failure is indeterminate, the tendons can be banded and the tendons can be draped resulting in beneficial vertical components of the vertical pre-stressing force. The purpose of this study is to predict punching shear resistance of pre-stressed concrete slab-column connection from physical analytical model, that includes integration of the vertical component of concrete tensile strength along the inclined surface of the inverted cone which represents the punching shear cracks, the dowel forces of supplementary bonded reinforcement and the pre-stressing force from pre-compression and vertical component of pre-stressing calculated by yield line theory. The comparisons of the present proposed equation with ECP 203, ACI 318, BS8810, and EC2 code provisions and Gardner equations for the previous experimental scaled studies indicated that the proposed equation is less conservative. Finally from the proposed equation the contribution of concrete tensile force, dowel force and pre-stressing force in the punching shear resistance of pre-stressed concrete flat slabs can be calculated.

Key words: Punching, Shear, Pre-stressed, Flat Slab

PREVISION DE BOXE RESISTANCE AU CISAILLEMENT DES DALLES EN BETON PRECONTRAINT PLAT

RÉSUMÉ:

Poinçonnement du béton précontraint dalle est un mode de défaillance indésirables qu'elle se produit sans avertissement et peut conduire à l'effondrement progressif de vastes zones de la dalle ou même des structures complètes. Modélisation de la résistance au cisaillement de poinçonnage de précontrainte des dalles de béton est conceptuellement difficile que les tendons peuvent être collées ou non liée, le tendon peut passer à travers les colonnes ou non, le stress dans les tendons à la rupture par poinçonnement est indéterminée, les tendons peuvent être bagués et les tendons peuvent être drapées entraînant bénéfique composante verticale de la force verticale de précontrainte. Le but de cette étude est d'étudier la prédiction de la résistance au cisaillement de poinçonnage de béton précontraint connexion dalle-colonne du modèle d'analyse physique, y compris l'intégration que de la composante verticale de la résistance à la traction en béton le long de la surface inclinée du cône inversé qui représente les fissures de poinçonnement, les forces de la cheville renforcements liés complémentaires et la force de précontrainte de la pré-compression et une composante verticale de la précontrainte du calcul de la théorie de la ligne du rendement. Les comparaisons de l'équation proposée présente avec ECP 203, ACI 318, BS8810, et les dispositions du code EC2 et les équations Gardner pour les études expérimentales échelle rapportés dans la littérature, ce qui indique que l'équation proposée est moins conservatrice. Enfin à partir de l'équation proposée la contribution de la force de traction en béton, la force de serrage et de précontrainte vigueur dans la résistance au cisaillement par poinçonnement du béton précontraint dalles peuvent être calculée.

MOTS-CLÉS: Poinçonnage, cisaillement, précontraint, dalles plates

---

* Received: 26/12/2010 , accepted: 15/1/2011

** Contact author: (e-mail: ragabshaker@yahoo.com.)

1. INTRODUCTION

Flat slabs are economical systems because that offers distinct advantages such as low cost due to the ease of construction (e.g., slip forms), low floor to floor height, and flexible use of space. In practical, pre-stressed flat slab systems are very efficient, since it provide improve crack and deflection control, and allow relatively large slab span-to-thickness ratios, on the order of 35 to 45 which allow increasing spans with reducing slab thickness. However, prestressed concrete and reinforced concrete flat slabs are susceptible to failure by punching shear.

Punching shear is undesirable mode of failures which occur without warning and can lead to progressive collapse of large areas of slab, or even of complete structures. Punching shear always occurs in regions of large moment, and flexural cracks are observed around the periphery of the loaded area or support.

2. ANALYTICAL MODEL

2.1 Punching Failure Mechanism

Punching is very complex failure mode and was for decades subject of many discussions and attempt of explications, the main point of its development are at present established [22] Fig. (1).

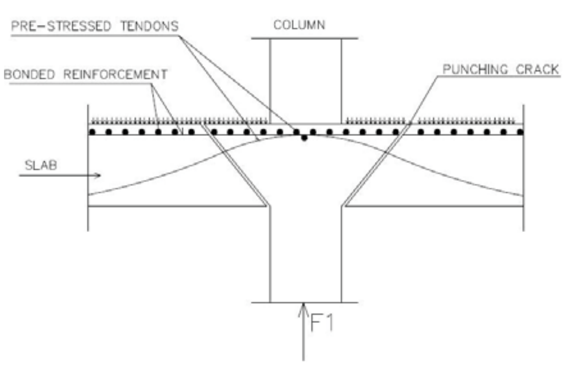

Fig. (1): Punching failure mechanism

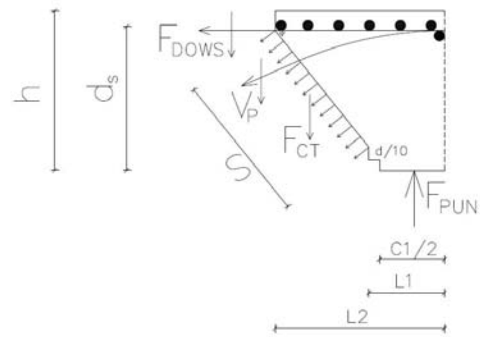

Fig.(2): Punching shear capacity

- Formation of a roughly circular crack around the column periphery on the tension surface of the slab and its propagation into the compression zone of the concrete.
- Formation of a new lateral and diagonal flexural crack.
- Initiation of an inclined shear crack near mid-depth of the slab, observed at about 1/2 to 2/3 of the ultimate load.
- With increasing loads the inclined cracks develop towards the compression zone, at failure, the punching crack has reached the corner of the slab-column intersection. Thus the punching strength is influenced by several material and geometrical parameters.

It is well known that the punching is controlled by cross section column size, slab depth and concrete strength as proposed by design codes [2, 9 & 10]. The influence of the percentage of flexural reinforcement (rebar ρs and pre-stressed ρp) are contribute as a factor of increasing tensile strength of concrete. While, from the examination of the punching failure mechanism, once of tensile stress around crack, the dowel action of used bonded reinforcement ($Fdows$) and the contribution of inclined shear crack has developed, the applied load is resisted by the vertical component pre-stressing steel ($Fpres$).

The proposed prediction equation for the punching shear strength of pre-stressed concrete flat slab-column connections, by extending the punching equation of reinforced concrete flat slab predicted by **Menetrey**[22].

Two parameters of concrete tensile strength and dowels contribution, but for contribution of pre-stressed both vertical components and initial decompression of concrete, these by extending of **Gardner**[13].

2.2 Proposed Model

By extending the analytical model developed by **Menetrey**[22]to compute the punching load of reinforced concrete slab, these model was based on the observation that the punching load is influenced by the tensile stress in concrete along the inclined punching crack. Consequently, the punching load is predicted by integration the vertical components of the tensile stress around the punching crack.

According to various codes of practices there are minimum bonded reinforcement positioned over column and bottom at mid-span i.e., in negative and positive moment areas, and then these supplementary bonded reinforcement contribute in resisting punching shear failure by dowels effect by taking summation of vertical components of the tensile forces of all bars crossing the punching cracks as illustrated in Fig. (2). The pre-stressing tendons passing through the punching cracks zone increasing punching shear strength of pre-stressed concrete flat slab.

Thus the total punching load of the pre-stressed concrete flat slab is obtained by:

Fpun = Fct + Fdows + Fpress (1)

Fct: The vertical component of the concrete tensile force calculates by integrating the vertical components of the tensile stress of concrete.

Fdows: The dowel force which is the vertical component of the tensile forces of all bars crossing the punching cracks.

Fpress: It is the initial decompression of concrete plus the vertical component.

2.2.1 Concrete Tensile Force (Fct)

The punching crack is assumed to be a truncated cone in shape developed between two lengths ***L1*** and ***L***2 as shown in fig.2 with:

$$L1=\frac{C1}{2}+\frac{d}{10\tan\alpha} \quad \text{and}$$

$$L2=\frac{C1}{2}+\frac{d}{\tan\alpha} \tag{2}$$

C: is the side dimension of the square column but for the circular and rectangular as square column in the same area.

C: is the side dimension of the square column but for the circular and rectangular as square column in the same area.

α : is the punching crack inclination, which is varying between 30degree and 60degree depending to the column cross-section and the slab depth of non pre-stressed reinforcement(***ds***).

The inclination length ***S*** of crack side is:

$$s=\sqrt{sq.(L2-L1)+sq.(0.9d)} \tag{3}$$

Assuming a constant tensile stress distribution around the punching crack, the analytical expression to compute the vertical component of the concrete tensile force is consequently expressed as follows:

$$\mathrm{Fct}=\left[2(\mathrm{C1}+\mathrm{C2})+\frac{2\pi d}{\tan\alpha}\right]S\sigma\mathrm{v} \tag{4}$$

$$\mathrm{Fct}=\left[2(\mathrm{C1}+\mathrm{C2})+\frac{2\pi d}{\tan\alpha}\right]S\mu\xi\eta\sqrt[3]{\frac{\mathrm{fcu}}{\gamma\mathrm{c}}} \tag{5}$$

Where: fcu; is the concrete compressive strength, factored to represent the tensile strength of concrete.

ξ ; is the factor taking into account the influence of the percentage of non pre-stressed reinforcement ρS determined numerically.

$$\xi = -0.1\rho s^2 + 0.46\rho s + 0.35 \geq 0.50 \quad (6)$$

where ρs in percent, and $0 \leq \rho s \leq 2$

μ : is the size effect law obtained numerically by **Shehata** [22]

$$\mu = 1.6(1 + \frac{ds}{da})^2 \geq 0.50 with (d \geq da) \quad (7)$$

da: is the aggregate size and ***ds*** is the slab depth of non pre-stressed reinforcement.

Thus by increasing ***da*** (or by decreasing ***ds***) the crack propagation through the slab thickness is limited and the concrete tensile force is increased.

In addition the size effect is influenced by the ratio of column side to slab depth, this phenomenon was reproduced numerically and analytically expressed as **Menetrey**[22].

$$\eta = 0.1(\frac{C}{2h})^2 - 0.5(\frac{C}{2h}) + 1.25 \ \& \ 0 \leq (\frac{C}{2h}) \leq 2.5$$

$$, \ \eta = 0.625 \ \& \ (\frac{C}{2h}) \geq 2.5 \quad (8)$$

The concrete tensile force depends up on the geometrical parameters, namely slab depth, column sides, compressive strength of concrete, percentages of steel reinforcement and the aggregates size.

2.2.2 Dowel Effect (Fdows)

Pre-stressed concrete flat slabs generally required to have supplementary bonded reinforcement for many purpose as temperature, shrinkage and even contribution as reinforcement in tension zones.

The uses of non pre-stressed steel over columns improve the behaviors of the slab-column connections and crack distribution.

In addition, the steel dowels effect interprets the amount of shear force that can be transferred by reinforcing bars crossing the punching crack.

$$\textbf{Fdows} = \frac{1}{2}\sum_{1}^{bars} \varphi s^2 \sqrt{fcfs(1-\zeta^2)} \sin\alpha \quad (9)$$

where:

$\frac{1}{2}\sum_{1}^{bars}$: is the summation of all bars crossing the punching crack.

$\varphi\ s$: is the diameter of non pre-stressed bars.

A parabolic interaction is assumed between the axial force and the dowel force in the reinforcing bar which is expressed with the term $(1 - \zeta^2)$ where $\zeta = \frac{\sigma s}{fs}$ and σs and fs is the axial tensile and yield stress in the non pre-stressed steel bars.

For pre-stressed concrete flat slab with a realistic percentage of flexural reinforcement less than one percentage and without shear reinforcement, the punching of slab occurred without yielding of the steel and tensile stress estimated to be 0.45 to 0.60 times the ultimate tensile strength.

The dowel contribution is reduced with term $\sin\alpha$ to take into account of the angle between the steel bar and the punching crack, the factor 1/2 gives the best approximation for horizontal steel bar which not crossing the punching crack at right angle.

Then the dowel force can be:

$$\textbf{Fdows} = \frac{1}{2}\sum_{1}^{bars} \varphi s^2 \sqrt{0.6 fcufys} \sin\alpha \ (10)$$

2.2.3 Pre-stressed Effect (Fprss)

For pre-stressed concrete flat slabs, the ratio of pre-stressed steel was calculated from the initial pre-compression at the column, i.e. $\rho p = \dfrac{fpc * h}{fse * dp}$.

The shear force ***Fprss***, appropriate to the decompression moment was calculated:

$$Fprss = 2\pi\rho p dp fps(dp - \frac{h}{3}) \quad (11)$$

where:

h : is the slab thickness in mm
dp: is the effective depth of slab for pre-stressed steel in mm.
fpc: the pre-compression strength in concrete calculated at cetroide of slab, MPa .
fps: the stress in pre-stressing steel at ultimate loads (assumed 1.1 fse), MPa
fse: the stress in pre-stress steel after all losses, MPa (developed from the yield line expression) for punching shear.

$Fyield = 2\pi M$; *M* is the yield moment per unit width [28] this avoids the problem of determining the inclination of the pre-stressing tendons crossing the failure surface required to calculate the vertical component of the pre-stressing force.

3. PUNCHING SHEAR OF PRE-STRESED CONCRETE FLAT SLAB

Punching shear has a position of special importance in the design of flat slabs, which are practically always under-reinforced against flexural, exhibit pronounced ductile bending failure.

In beams, due to the usual present shear reinforcement, a ductile failure is usual assured in shear also, since slabs by contrast, are provided with punching shear reinforcement is avoided of at all possible for practical reasons. Punching shear is associated with a brittle failure of the concrete. In practical discussion how the prestress can be taken into account in the existing design specification which have usual been developed for ordinary reinforced flat slabs.

In the last years, many design formula have been developed, which where obtain from empirical investigations and a few practical cases, by model representation. Many codes have developed empirical relationships for punching shear capacities. This based on the concrete strength and critical section perimeter located at a specific distance from column face.

3.1 Egyptian Code of Practices [11]

The Egyptian Code of Practice (ECP-203) (11) introduced an equation to calculate the punching shear capacity of two-way pre-stressed concrete flat slab:

$$q_{cup} = \beta p\sqrt{\frac{fcu}{\gamma c}} + 0.1 fpcc + fpv \ \mathrm{N/mm^2} \quad (12)$$

where :

$\beta\ p$ = least value of $= 0.275$

$$= 0.8\left[0.15 + \frac{ab}{b}\right] \qquad o$$

f_{pcc} = average of concrete strength at the perimeter of the critical section (after pre-stressing losses) at the middle of thickness of the slab.

q_{pv} = shear strength due the vertical component of the pre-stressing force The last equation is used only for interior column, while the specified concrete cylinder strength taken not is greater than ***40 MPa*** and the average value of the pre-compression for two directions at the centroid of concrete shall not be less than ***0.9 MPa*** or greater than ***3.5 MPa***.

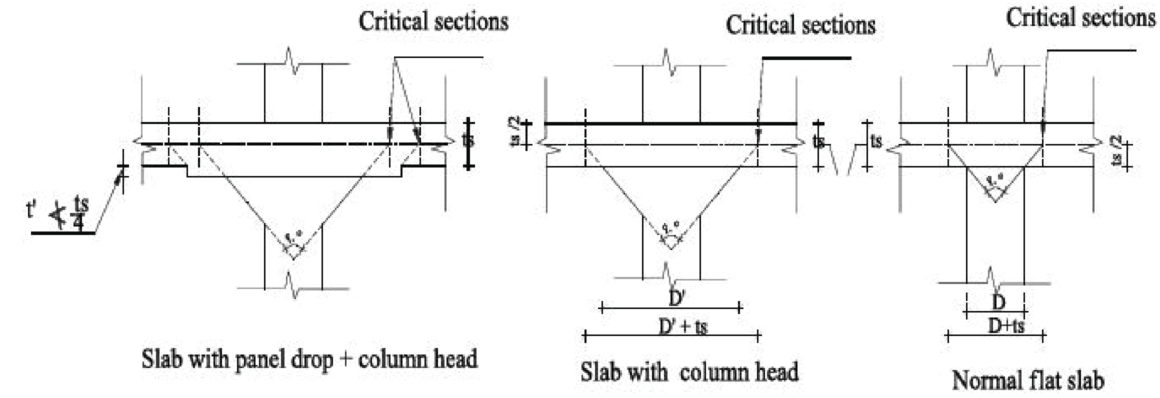

Fig. (3): Critical sections for punching (ECP-203) [10]

3.2 American Code Institute ACI-318[1 & 2]

ACI-318-2008 [1] specifies that the shear capacity should be calculated on the minimum perimeter located at a distance ***d/2*** from the periphery of the column or the concentrated load. These provisions follow from the work of ACI-ASCE Committee 423 [3].

The punching shear strength around the interior columns of two-way pre-stressed concrete flat slab can be predicted by:

$$V_n=(\beta p\sqrt{fcu}+0.3f_{pc})b_o d+V_p \quad (13)$$

where :

b_o = π (c + d) for circular loaded areas, mm.

b_o = (4b + d) for square loaded areas, mm

d = effective slab depth to reinforcement. Mm.

c = diameter for circular loaded area, mm

However, ***d*** need not be taken as less than ***0.8h*** for pre-stressed concrete flat slab.

For rectangular columns a critical section with four straight sides can be used.

f_{ck} = the specified concrete cylinder strength, *MPa*. (not be taken more than ***35 MPa***.)

V_n = nominal shear strength, N.

αs = 3.30 for interior column, 2.50 for edge column, 1.67 for corner column.

βp = the smaller of 0.29 or ($\frac{\alpha sb}{bo}$ + 0.13)

f_{PC} = the average value of the pre-compression for two directions at the centroide of slab, it shall not be less than ***0.9 MPa*** or greater than ***3.5 MPa*** for each direction.

V_P = the vertical component of all effective pre-stressing forces crossing the Critical section.

h = overall slab thickness.

3.3 BS8110-97 [6]

The British Code [6] used a rectangular control perimeter at 1.5d from the loaded area for both circular and rectangular loaded areas. For reinforced concrete flat slabs the available shear force can be calculated from:

$$\frac{V_{eff}}{b_0 d}<v_{CBS}=\frac{0.79}{\gamma_m}\left(100\rho\frac{f_{cu}}{25}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}<0.8\sqrt{f_{cu}}<5.0 \quad MPa$$

where:

γ_m = partial safety factor for strength of material (1.25).

f_{cu} = the specified concrete cube strength(taken as 1.25 f_{ck}) Mpa.

b_o = 4(c+3d) for circular loaded areas, mm.

b_o = 4(c+3d) for square loaded areas, mm

ρ = (ρ_x + ρ_y)/2 calculated for a width equal to (c+3d) or (b+3d).

ρ_x = ratios of reinforcement in the X direction.

ρ_y = ratios of reinforcement in the y direction

BS811-97[6]

The handbook of the British standard [6] suggests that the punching shear capacity of slabs with bonded tendons can be calculated as for an ordinary reinforced concrete flat slab using the actual area of tendons, and to add the decompression load; the load required to annul the pre-stress in the extreme fibers put into tension by the applied loading. For unbonded tendons the punching shear resistance would be 10 percent less. This method was first developed by Regan [20] for one-way bonded pre-stressed concrete slabs, and was extended to two-way slabs.
For corner columns and edge columns with bending about an axis parallel to the free edge, the punching shears resistance is taken by:

$$V_{eff} = 1.25\, V_u \qquad (15)$$

3.4 Euro Code 2[9 & 10]

According to Euro Code2 [9 & 10] the principals, rules and design model for checking punching shear failure at the ultimate limit state are shown in Fig. 4. The shear resistance shall be checked along a defined critical perimeter. In slabs subjected to punching shear an enhancement of shear resistance should not be carried out. If the thickness of a slab is not sufficient to ensure adequate punching shear resistance, shear reinforcement, column heads or other types of shear connector shall be provided. These rules also apply to waffle slabs with a solid section around the loaded area provided that the solid area extended at least ***1.5d*** beyond the critical perimeter.
The amount of longitudinal tensile reinforcement in two perpendicular directions, x and y should be greater than 0.5 percent, calculating allowing for any differences in effective depth in the two directions. The force component parallel to V_{sd} due to inclined pre-stressed tendons placed inside the critical area may be taken into account [8 & 9].

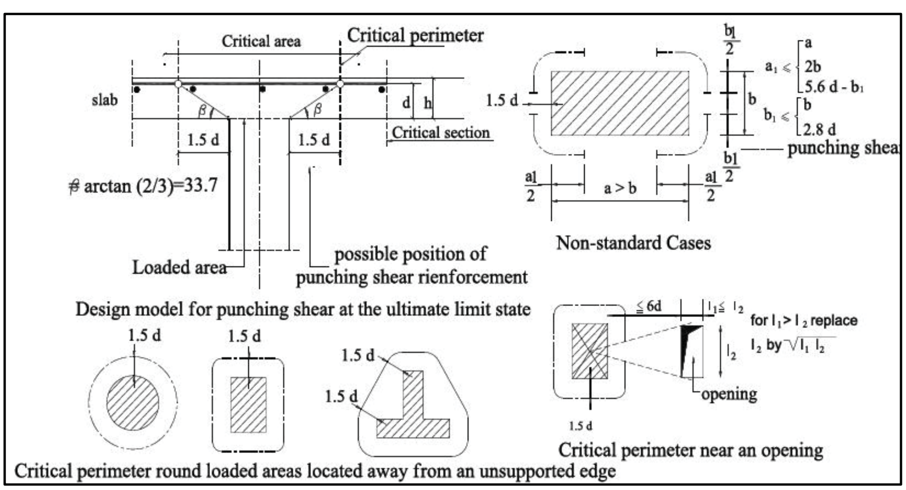

Fig.(4): Principals and design model for punching shear (8 & 9)

The shear resistance per unit length τ_{Rd} of pre-stressed slab is given by:

$$\boldsymbol{v_{rd1} = \tau_{Rd}\, k(1.2+40\,\rho_1)d} \qquad (16)$$

where τ_{Rd} *is given in . Euro code 2-section 4.3.2 .*

$k = |(1.6-d) \geq 1.5|$ (d in meters)

$$\rho_l = \sqrt{\rho_{lx} . \rho_{ty} + \frac{\sigma_{tpo}}{f_{yd}}} \leq 0.015 \qquad (17)$$

ρ Lx and ρ 1y relate to the tension in X and y direction respectively.

$d = (dx + dy)/2$

d_x and d_y are the effective depths of the slab at the points of intersection between the design failure surface and the longitudinal reinforcement in x and y directions respectively.

$\sigma_{cpo} = N_{pd} / A_c$

$\boldsymbol{f_{yd}}$ = design yield stress of the reinforcement .

N_{pd} = pre-stressing force corresponding to the initial value without losses. If the pre-stressing force is differed in the two directions, then the average value is used. N_{pd} should be calculated with γ =0.9.

3.5 Gardner Method [14]

Gardner [14] proposed an equation for predicting the punching shear strength of interior slab-column connection of reinforced and pre-stressed concrete flat slabs, by extending the work of **Shehata** and **Regan** [27] and **Shehata**[26]. **Gardner**(14) examined the dependence of the punching shear strength to the concrete strength and the strength for reinforced and pre-stressed concrete slabs using control perimeter at the periphery of the loaded area and **Shehat** [26] type strength enhancement expression.

Columns with circular or rectangular cross sections were analyzed as square columns of the same cross sectional area. A sensitivity analysis, using the coefficient of variation of the equation which is the criterion of goodness, was used to confirm that the one-third power of concrete strength and steel forces was close to optimal. For unbonded post-tensioned slabs, the pre-stressed reinforcement ratio was calculated from the initial pre-compression at the column. i.e., $\rho_p = f_{pc}h/f_{se}d_p$

The shear force $\boldsymbol{Vd}$,appropriate to the decompression moment, was calculated by; $\boldsymbol{V_d \cong 2\pi\rho_p d_R f_{ps}(dp-h/3)}$ (developed from the yield line expression for punching shear; $V_{yield}=2\pi M$; $\boldsymbol{M}$ is the yield moment per unite width [32] . This avoids the problem of determining the inclination of pre-stressing tendons crossing the failure surface required to calculate the vertical component of the pre-stressing force. For design purpose the decompression shear force is multiplied by ***0.75*** to represent the lower 95 percent bound.

$$V_c = 0.55\lambda u d_{eff}\left[1+\left(\frac{250}{h}\right)^{0.5}\right]\left(\frac{h}{4c}\right)^{0.5}\times\left(\frac{\rho f_y d^3+\rho_p f_{ps} d_p^3}{d_{eff}^3}\right)^{1/3}\left[f_{ck}^{(1/3)}\right]+0.75V_d\left[\frac{u}{4c}\right] \qquad (18)$$

$\left[1+\left(\frac{250}{h}\right)^{0.5}\right]$ = size effect term.,

$\left(\frac{h}{4c}\right)^{0.5}$ = strength enhancement factor

c = dimension of square column of same cross sectional area as the actual column, mm

d = effective slab depth to reinforcement, mm.

d_p = effective slab depth to pre-stressed reinforcement, mm.

$\boldsymbol{d_{eff}}$ $=(\rho f_y d^2+\rho_p f_{se} d_p^2)/((\rho f_y d+\rho_p f_{se} d_p)$

fck = specified concrete cylinder strength' MPa.
fse = effective stress in pre-stressed reinforcement after
f_{pc} = pre-compression in concrete calculated at the centroid of the slab, MPa.
f_{ps} = stress in pre-stressed reinforcement at ultimate load. (assumed =1.1***fse***).
f_y = yield strength of flexural reinforcement, in MPa.
h = slab thickness, mm.
λ = 1 for normal density concrete. 0.85 for light weight concrete.
U = perimeter of equivalent square column attached to slab, mm.
ρ = ratio of flexural tensile reinforcement calculated over a width c + 6d
ρ_p = ratio of per-stressed steel calculated from ρ p = f***pc h*** / ***fse dp***

4. COMPARISON OF PREDICTION METHODS:

This section compares the capacities predicted by ECP-203 [11], ACI-318 [1], BS-8110 [6], EC2 [10], Gardner [14] proposed equation and the proposed equation explained at analytical model with the measured capacities obtained from experimental studies reported in the next tables shown below.

Because code expressions are written in terms of specified strength, and experimental investigations report mean strength, the code calculated values were calculated using a factored concrete strength *(0.8fcu)*. The pre-stress strength (*fps**)*** at punching failure was measured to be *110* percent of the effective pre-stress stress (*fse**)***. The comparisons are presented using slab depth *(d)* equal 0.8 the slab thickness ***(h)*** with a rectangular shear perimeter and taking into consideration the effect of vertical pre-stress component.

4.1 Interior Column-Slab Connections With Supplementary Bonded Reinforcement:

Table (1, 2, 3 and 4) gives the experimental data and result punching failure force for previous studies.

Table (1): Data and results of experimental studies by Frrankline/Long [13] , Smith/Burns [31], Burns [7], Koust [19], Razi [23], Scordel/Lin [30], and Gerber/Burns [15]

Specimen	*fcu MPa*	ρ percent	*fy MPa*	ρp percent	*fse MPa*	*fpc MPa*	*h mm*	*d* mm	*dp mm*	*Column size*	*Vtest KN*
Frrankline/Long(13)	38	0.72	517	0.33	1020	2.43	60	44	44	170	128
Smith/Burns(31) S2	29	0.63	400	0.38	759	2.24	70	54.4	54.4	203	121.5
Smith/Burns(30) S2	32	0.94	400	0.38	759	2.24	70	56.4	54.4	203	135.3
Burns(7) SI 5	34	0.46	400	0.36	768	2.24	70	46.1	58.3	203	128.6
Burns(7) SI 6	34	0.46	400	0.36	768	2.24	70	60.2	58.3	203	152.6
Burns(7) SI 7	34	0.46	400	0.36	768	2.24	70	60.2	58.3	203	157
Burns(7) SI 9	34	0.46	400	0.36	768	2.24	70	60.2	58.3	203	162.4
Burns(7) SI 10	34	0.46	400	0.36	768	2.24	70	60.2	58.3	203	125
Burns(7) SII 10	33	0.56	400	0.09	1220	0.93	70	60.2	59.1	203	99.2
Koust(19) S3-5	27	0.60	420	0.16	936	1.24	70	60.2	59.2	203	120
Razi(23) 9	29	0.45	490	0.26	1040	2.22	165	138	138	300	564
Razi(23) 10	29	0.45	490	0.15	1040	1.28	165	138	138	300	416
Razi(23) 4	29	0.06	490	0.12	1040	0.61	200	169	100	300	634
Scordel/Lin(30)	41	0.07	450	0.12	1054	1.05	76	63.5	63.5	229	120
Gerber/Burns(15) C2	32	0.40	430	0.30	986	2.35	178	140	140	300	857
Gerber/Burns(15) C3	34	0.40	590	0.30	986	2.35	178	140	140	300	901

Table (2): Data and results of experimental studies by Silva [28]

Speci-men	*fcu MPa*	ρ *percent*	*fy MPa*	ρp *percent*	*fse MPa*	*fpc MPa*	*h mm*	*d mm*	*dp mm*	*Column size*	*Vtest KN*
A1	37.8	0.62	553	0.45	1009	3.31	125	109	91	100sq	380.0
A2	37.8	0.47	553	0.28	1009	2.14	127	113	97	100sq	315.6
A3	37.8	0.62	553	0.47	1009	3.16	128	109	86	100sq	352.7
A4	37.8	0.51	553	0.29	1009	1.98	129	104	86	100sq	321.0
B1	40.1	0.6	553	0.43	1009	3.39	124	114	98	200sq	582.5
B2	40.1	0.48	553	0.29	1009	2.23	124	110	94	200sq	488.0
B3	40.1	0.63	553	0.43	1009	3.12	124	108	90	200sq	519.8
B4	40.1	.50	553	0.30	1009	2.16	124	106	89	200sq	458.8
C1	41.6	0.61	525	0.44	1009	3.33	126	111	94	300sq	720.0
C2	41.6	.50	525	0.31	1009	2.26	122	105	89	300sq	556.1
C3	41.6	0.64	525	0.48	1009	3.48	124	106	90	300sq	636.6
C4	41.6	0.52	525	0.33	1009	2.31	123	102	85	300sq	497.1
D1	44.1	0.68	540	0.50	1009	3.34	124	100	83	200sq	497.1
D2	44.1	0.50	540	0.30	1009	2.23	123	106	90	200sq	385.2
D3	44.1	0.51	540	0.31	1009	2.27	125	103	90	200sq	395.2
D4	44.1	0.48	540	0.29	1009	2.22	125	111	95	200sq	561.5

Table (3): Data and results of experimental studies by Correa [8], Kordina and Nolting [18], Hassavzadah [16], Shehata [25], and Melges [21]

Specimen	*fcu MPa*	ρ *percent*	*fy MPa*	ρp *percent*	*fse MPa*	*fpc MPa*	*h mm*	*d mm*	*dp mm*	*Column size*	*Vtest KN*
correa(8)LP2	52.4	1.17	550	0.43	900	2.19	130	105	65	150sq.	355
correa(8)LP3	52.6	1.17	550	0.85	900	4.28	130	105	65	150sq.	415
correa(8)LP4	50.7	1.17	550	0.13	900	0.80	130	105	81	150sq.	390
correa(8)LP5	50.7	1.17	550	0.20	900	1.33	130	105	81	150sq.	475
correa(8)LP6	51.4	1.17	550	0.28	900	1.76	130	105	81	150sq.	437
Koodina and Nolt-ing(18)V1	33.6	0.62	500	0.22	920	1.70	150	128	113.5	200cir.	450
Koodina and Nolt-ing(18)V2	36.0	0.90	500	0.22	920	1.66	150	126	113.5	200cir.	525
Koodina and Nolt-ing(18)V3	36.0	0.62	500	0.41	920	3.09	150	128	113.5	200cir.	570
Koodina and Nolt-ing(18)V7	31.2	0.62	500	0.23	920	1.77	150	128	113.5	200cir.	475
Koodina and Nolt-ing(18)V8	35.2	0.62	500	0.26	920	1.77	150	128	113.5	200cir.	518
Hassavzadah(16)A1	31.0	0.18	460	0.33	950	2.79	180	150	151	250cir.	668
Hassavzadah(16)A2	28.7	0.18	460	0.34	950	2.74	180	150	144	250cir.	564
Hassavzadah(16)B2	43.8	0.29	460	0.25	950	2.12	220	190	110	250cir.	827
Hassavzadah(16)B3	41.1	0.29	460	0.26	950	2.21	220	190	191	250cir.	1113
Hassavzadah(16)B4	43.2	0.29	460	0.23	950	1.99	220	190	190	250cir.	952
Shehata(25)SP1	36.5	0.27	530	0.51	918	3.94	175	157	135	150sq.	988
Shehata(25)SP4	41.7	0.27	530	0.62	918	4.81	175	157	135	150sq.	884
Shehata(25)SP5	40.9	0.27	530	0.43	918	3.28	175	157	135	150sq.	780
Shehata(25)SP6	42.5	0.27	530	0.45	918	3.50	175	157	135	150sq.	728
Melges(21)M4	51.9	0.92	550	0.26	925	1.95	160	134	120	180sq.	773

Table (4): Data and results of experimental studies by A. Mousa [4], and A. Pinho Romas [5]

Specimen	*fcu MPa*	ρ *percent*	*fy MPa*	ρp *percent*	*fse MPa*	*fpc MPa*	*h mm*	*d mm*	*dp mm*	*Column size*	*Vtest KN*
A.MousaPFS1(4)	34.7	0.77	500	0.29	800	2.17	80	64	64	180sq.	124
A.MousaPFS2(4)	37.8	0.77	500	0.29	800	2.17	80	64	64	180sq.	110
A.MousaPFS3(4)	33.1	0.77	500	0.29	800	2.17	80	64	64	180sq.	115
A.MousaPFS4(4)	37.2	0.77	500	0.29	800	2.17	80	64	64	180sq.	120
A.MousaPFS6(4)	38.6	0.77	500	0.29	800	2.9	80	64	64	180sq.	134
A.MousaPFS9(4)	33.7	0.77	500	0.38	800	2.17	80	64	64	180sq.	146
A.PinhoomasAR8(5)	52	0.59	450	0.34	900	2.70	100	80	80	200sq.	308
A.PinhoomasAR10(5)	51.8	0.59	450	0.34	900	2.70	100	80	80	200sq.	315
A.PinhoomasAR11(5)	47.5	0.59	450	0.34	900	2.70	100	80	80	200sq.	302
A.PinhoomasAR12(5)	39.1	0.59	450	0.34	900	2.70	100	80	80	200sq.	280
A.PinhoomasAR15(5)	39.6	0.59	450	0.34	900	2.70	100	80	80	200sq.	262

Table (5): Comparison of prediction methods for experimental studise by Frrankline/Long [13], Smith/Burns [31], Burns [7], Koust [19], Razi [23], Scordel/Lin [30], and Gerber/Burns [15]

Specimen	V_{test}/V_{ECP203}	V_{test}/V_{ACI}	V_{test}/V_{BS8110}	$V_{test}/V_{Gardner}$	$V_{test}/V_{Proposed}$
Frrankline/Long(13)	1.50	1.28	1.22	1.18	0.91
Smith/Burns(31) S2	1.15	0.98	0.89	0.93	0.95
Smith/Burns(31) S2	1.24	1.06	0.91	0.96	0.98
Burns(7) SI 5	1.16	0.99	1.51	0.91	1.02
Burns(7) SI 6	1.35	1.17	1.05	1.09	1.13
Burns(7) SI 7	1.4	1.20	1.08	1.12	1.16
Burns(7) SI 9	1.45	1.24	1.12	1.16	1.20
Burns(7) SI 10	1.12	0.96	0.86	0.89	0.93
Burns(7) SII 10	1.11	0.95	0.87	0.88	0.87
Koust(19) S3-5	1.35	1.17	0.96	0.99	1.00
Razi(21) 9	1.35	1.16	1.07	1.07	1.30
Razi(23) 10	1.17	1.00	0.97	0.97	0.98
Razi(23) 4	1.60	1.35	1.50	1.48	1.30
Scordel/Lin(30)	0.90	0.77	1.05	0.95	0.97
Gerber/Burns(15) C2	1.70	1.50	1.51	1.51	1.27
Gerber/Burns(15) C3	1.80	1.55	1.57	1.52	1.29

Table (6): Comparison of prediction methods for test results of experimental studies by Silva [28]

Specimen	V_{test}/V_{ECP203}	V_{test}/V_{ACI}	V_{test}/V_{EC2}	$V_{test}/V_{Proposed}$
A1	1.86	1.54	1.09	1.05
A2	1.62	1.35	0.99	1.01
A3	1.81	1.51	1.13	1.08
A4	1.92	1.60	1.25	1.18
B1	1.67	1.39	1.20	1.29
B2	1.65	1.37	1.26	1.33
B3	1.73	1.44	1.26	1.15
B4	1.73	1.44	1.28	1.20
C1	1.62	1.35	1.29	1.14
C2	1.49	1.24	1.28	1.25
C3	1.55	1.29	1.28	1.16
C4	1.4	1.18	1.20	1.13
D1	1.67	1.39	1.26	1.17
D2	1.39	1.16	1.05	1.0
D3	1.5	1.25	1.15	1.04
D4	1.30	1.08	1.13	1.06

Table (7): Comparison of prediction methods for results of experimental studies by Ccorrea [8], Kordina and Nolting [18], Hassavzadah [16], Shehata [25], and Melges [21]

Specimen	V_{test}/V_{ECP203}	V_{test}/V_{ACI}	V_{test}/V_{EC2}	$V_{test}/V_{Proposed}$
correa(8)LP2	1.70	1.41	0.92	0.98
correa(8)LP3	1.70	1.41	0.97	1.04
correa(8)LP4	2.15	1.80	1.05	1.12
correa(8)LP5	2.40	2.0	1.21	1.30
correa(8)LP6	2.10	1.76	1.06	1.17
Koodina and Nolt-ing(18)V1	1.56	1.30	0.93	1.02
Koodina and Nolt-ing(18)V2	1.85	1.54	1.02	1.13
Koodina and Nolt-ing(18)V3	1.90	1.58	0.96	1.06
Koodina and Nolt-ing(8)V7	1.67	1.39	0.99	1.08
Koodina and Nolt-ing(18)V8	1.74	1.45	1.04	1.08
Hassavzadah(16)A1	1.58	1.32	1.25	1.08
Hassavzadah(16)A2	1.58	1.29	1.28	0.97
Hassavzadah(16)B2	1.74	1.45	1.30	1.25
Hassavzadah(16)B3	1.97	1.64	1.29	1.25
Hassavzadah(16)B4	1.88	1.57	1.29	1.10
Shehata(25)SP1	2.40	2.00	1.66	1.50
Shehata(25)SP4	2.10	1.78	1.38	1.18
Shehata(25)SP5	2.00	1.68	1.58	0.97
Shehata(25)SP6	1.83	1.53	1.38	1.16
Melges(21)M4	2.34	1.95	1.31	1.40

Table (8): Comparison of prediction methods for results of experimental studies by A. Mousa [4], and A. Pinho Romas [5]

Specimen	Vtest/VECP203	Vtest/VACI	Vtest/VBS8110	Vtest/VGardner	Vtest/VProposed
A.Mousa PFS1(4)	1.50	1.26	1.18	1.21	0.97
A.Mousa PFS2(4)	1.28	1.07	1.02	1.01	0.90
A.Mousa PFS3(4)	1.56	1.30	1.20	1.15	1.03
A.Mousa PFS4(4)	1.44	1.20	1.15	1.10	1.08
A.Mousa PFS6(4)	1.76	1.47	1.41	1.35	1.12
A.Mousa PFS9(4)	1.60	1.29	1.23	1.26	1.20
A.Pinho omasAR8(5)	2.00	1.48	1.40	1.35	1.06
A.Pinho oma-sAR10(5)	1.74	1.45	1.35	1.30	1.08
A.Pinho oma-sAR11(5)	1.80	1.50	1.41	1.25	1.06
A.Pinho oma-sAR12(5)	1.60	1.40	1.31	1.18	1.03

Tables (5, 6, 7 and 8) give the Comparison of results of experimental studies reported in tables (1, 2, 3 and 4).

Table (9) summarizes the average percentages of the test load by the code provisions load of the ECP 203-07, ACI 318-05, BS 8110-94, EC2-92, Gardner 1998, and the proposed equation for the interior pre-stressed concrete flat slab-column connections.

Table (9): Comparison of the averages of the ECP 203-07, ACI 318-05, BS 8110-97, EC2-92, Gardner1998, and the proposed equation

Code	ECP203-07	ACI 318-05	EC2-92	BS 8110-97	Gardner	proposed
Average	1.65	1.37	1.19	1.15	1.14	1.12

The proposed equation is the less conservative to prediction the punching shear resistance of the pre-stressed concrete flat slab where it simulate the connection equilibrium under summation of the vertical forces.

The forces which contributed in the resistance of punching shear from the proposed equation taking into consideration approximately the all factors affecting on the punching shear failure.

The tensile concrete forces which contribute by approximately ***65 percent*** in the resistance of punching shear, which is the function of column side and slab depth as code provisions critical perimeter, the aggregate maximum nominal size and column slab thickness ratio also affected the tensile force of concrete, the percentage of non pre-stressed reinforcement contributing in increasing the concrete tensile force which is affected the concrete ductility.

The dowels of non pre-stressed reinforcement which crossing the punching crack contributing by ***11 percent*** in resisting punching shear strength, whereas the pre-stressed reinforcement contribute by ***24 percent*** to resist the punching shear as pre-compression of concrete and vertical component of pre-stressing crossing the punching crack, but calculating by the yield line formula presented previously.

4.2 Exterior and Corner Slab Column Connections With Supplementary Bonded Reinforcement

All the prediction equations are sensitive to the pre-compression at the critical section.

For the exterior slab-column connections the pre-compression, depth to the tendons, and the pre-compression shear are different on the various sides of the connection. For edge columns, punching shear failure usually initiates on the inside face of the connection, and hence, the pre-compression and the depth to the steel on the interior face control the failure.

For exterior and corner columns, this gives pre-compression values considerably larger than those reported in the original literature and probably larger than existed in the tested slabs. Estimating the depth to the tendons at the inside faces of a column, which would be less than the depth at the anchor due to tendon drape, is subjected to uncertainty.

The tables shown below are summarized the result data and the comparisons of code provisions and the proposed equation with decreasing the concrete tensile force to half for exterior connections and quarter for corner connections.

Table (10): Data and results of experimental studies by Long [20], Burnes [7], and Kousut [19]

Specimen	*fcu* *MPa*	ρ *percent*	*fy* *MPa*	ρp *percent*	*fps* *MPa*	*fpc* *MPa*	*h* *mm*	*d* *mm*	*dp* *mm*	*Column size*	*Vtest* *KN*
Long(20) E1	39	1.00	517	0.78	880	4.40	50.8	44	25.4	163sq	46.7
Long(20)E2	39.7	0.67	517	0.98	990990	7.00	50.8	44	25.4	163sq	49.8
Long(20)E3	39.4	0.67	517	1.59	990	10.6	50.8	44	25.4	163sq	56
Long(20)E4	39.6	0.67	517	0.49	990	3.5	50.8	44	25.4	163sq	40.9
Long(20)E5	36.06	0.67	517	0.71	990	7.00	50.8	44	33	163sq	53
BurnsII 2(7)	32.75	0.6	400	0.31	1220	1.90	70	60.2	34.9	203sq	30.7
Koust S3-6(19)	27.2	0.6	420	2.87	936	13.4	70	60.2	34.9	178sq	50
Koust S3-8(19)	27.2	0.6	420	1.00	936	4.65	70	60.2	34.9	203sq	75
Koust S3-2(19)	27.2	0.6	420	1.52	936	7.00	70	60.2	34.9	203sq	72.9

Table (11) Comparison of prediction methods for results of experimental studies by Long [20], Burnes [7], and Kousut [19].

Specimen	V_{test}/V_{ACI}	V_{test}/V_{BS8110}	$V_{test}/V_{Gardner}$	$V_{test}/V_{Proposed}$
Long(20) E1	1.31	1.10	0.98	1.02
Long(20)E2	1.24	1.11	1.01	1.04
Long(20)E3	1.13	1.04	1.00	1.03
Long(20)E4	1.30	1.13	0.98	0.99
Long(20)E5	1.12	0.90	0.90	1.13
BurnsII 2(7)	0.73	0.65	0.57	0.52
Koust S3-6(19)	0.62	0.55	0.57	0.63
Koust S3-8(19)	1.47	1.32	1.20	1.25
Koust S3-2(19)	1.19	1.09	1.03	1.04

From the proposed equation of resisting punching shear is ***60 percent*** contribution by concrete tensile force, ***32 percent*** contribution by pre-stressing force, and ***8 percent*** contribution by non pre-stressed reinforcement dowel forces.

7. CONCLUSIONS

From analytical model of proposed equations, the following is concluded that:

1. The tensile concrete forces which contribute by approximately ***61.7 percent*** in the resistance of punching shear, which is the function of column side and slab depth as code provisions critical perimeter, the aggregate maximum nominal size and column slab thickness ratio, the percentage of non pre-stressed reinforcement contributes in increasing the concrete tensile force which is affected the concrete ductility.
2. The dowels of non pre-stressed reinforcement crossing the punching crack contribute by ***11. 7 percent*** in resisting punching shear strength.
3. The pre-stressed reinforcement contribute by ***28.23 percent*** in resisting the punching shear, it is the summation of the pre-compression of concrete and vertical component of pre-stressing steel crossing the punching crack.

For the exterior and corner connections the resisting forces are ***60, 8, and 32*** percent for concrete tensile force, dowel force and pre-stressing force.

Table (12): Comparison of the averages of the ECP 203-2007, ACI 318-2008, BS 8110-97, EC2-92, Gardner 1998, and the proposed equation

Code	ECP203	ACI318	EC2	BS8110	Gardner	proposed
Average Interior	1.65	1.37	1.19	1.15	1.14	1.12
Average Ext & Corner	1.25	1.12	1.07	0.99	0.92	0.96

1. Table (12) which represents the comparison of the averages results obtained from codes ECP 203-07, ACI 318-05, BS 8110-97, EC2-92, Gardner 1998, and the proposed equation for interior, exterior and corner pre-stressed concrete flat slab.
2. The proposed equation is the main conclusion of the present work is:
   Fpun = Fct + Fdows + Fpress

$$Fct=\left[2(C1+C2)+\frac{2\pi d}{\tan\alpha}\right]S\mu\xi\eta\sqrt[3]{\frac{fcu}{\gamma c}},$$

$$\textbf{Fdows}=\frac{1}{2}\sum_{1}^{bars}\varphi s^{2}\sqrt{0.6fcufys}\,\sin\alpha,$$

$$Fpress=2\pi\rho pdpfps(dp-\frac{h}{3})$$

REFERENCE

1- **ACI Committee 318 (2005)**, Building Code Requirements for Concrete (ACI 318-05) and COMMENTARY (318r-05), American Concrete Institute, Farmington Hills, MI, 2005, 430PP.

2- **ACI Committee 318 (2008)**, Building Code Requirements for Concrete (ACI 318-0٨) and COMMENTARY (318-05), American Concrete Institute, Farmington Hills, MI, 2008, Chapter 18.

3- **ACI-ASCE Committee 423,** Recommendations for Pre-stressed Concrete Members with Unbonded Tendons, ACI Structural Journal, V. 86, No. 3, May-June 1889, PP. 301-318.

4- **A. Moussa, and A. Salama.,** "Punching Shear Behaviour of Prestressed Flat Slabs," CSCE Annual Conference, June 10-13, 2008.

5- **A. Pinho Ramose and V. J. G. Lucio.,** "Post-Punching Behaviour of Prestressed Concrete Flat Slabs," Magazine of Concrete Research, V. 60, No. 4, May 2008, PP. 245-251.

6-**BS8110., Structural Use of Concrete;** Part 1: Code of Practice for Design and Construction. British Standards Institution, London, 1997.

7- **Burns, N. H., and Hemakon, R.,** Test of Scale Model Post-Tensioned Flat Plate, Journal of Structural Division, ASCE, v. 103, No. ST6, June 1977, PP. 1237-1255.

8- **Correa, G. S.,** "Puncionamento em Lajes Cogumelo Protendidas com Cabos Nao Aderentes", MSc dissertation, Department of Civil and Environmental Engineering, University of Brasilia, Brasilia, 200١, 103PP.

9- **Eurocode 2**, "Unbonded and External Prestressing Tendons," Part 1-5 ENV, 1992.

10- **Eurocode 2,** "Design of Concrete Structures," ENV 1992-1-1 Dec. 1991.

11- **ECP 203 (2007), Egyptian Code for Design and Construction of Concrete Structures**, Permanent Committee of the Egyptian Code for Design and Construction of Concrete Structures, Housing and Building Research Center, Ministry of Housing, Utilities and Urban Communities, Giza, Egypt.

12- **Elsalakawy EF, Polla MA, Soudi KA**."Relation of Reinforced Concrete Slab-Clumn Connections." Can. J. Civil. Eng. 2002;29; 602-u.

13- **Franklin, S. O., and Long, A. E.,** "The Punching Behaviour of Unbonded Post-Tensioned Flat Plates," Proc. Instm Civ. Engrs, Part.2, 73, Sept. 1982, PP. 609-931.

14- **Gardner, N. J.,** "Punching Shear Provision for Reinforced and Prestressed Concrete Flat Slabs," CSCE Journal, V. 23, No. 2, Apr 1996, pp. 502-510.

15- **Gerber, L.L., and Burns, N. H.,** Ultimate Strength Tests of Post-Tensioned Flat Plates, Journal of Prestressed Concrete Institute, V. 16, No. 6, Nov. 1971, PP. 40-58.

16- **Hasssanzadeh, G.**, "Petog Plattor pa Pelare, Dimensioenigs Metoder for Platter Med Ickevidh a Ftande Spannarmering," TRITA – BKN Bulletetin 43, Institutionen for Byggkonstuktion, Kungl – Tekniska Hogskolan, Stockholm, Sweden, 1998, PP. 100-130.

17- **Harianto Sundidja and Urban Illions.,** "Response of Prestressed Concrete Plate-Edge Column Connections," PhD Thesis, 1982.

18- **Kordina, K., and Nolting, D.,** "Versuche Zum Durchstanzen ohne verbund Vorgespannter Flachdecken", Lehrtuhl fur Massivbau, Technische Universitat, Braunschweig, Germeny, 1984,59PP.

19- **Koust. G. M.,** Shear Strength of Post-Tensioned Concrete Flat Plate at the Column Connections., MSc Thesis, University of Texas and Austin, June, 1977.

20- **Long, A. E., and Cleland, D. J.,** "Post-Tensioned Flat Slabs at Edge Columns," ACI Journal, V. 90, No. 3, May-June 1993, PP. 207-213.

21- **Melges, J. L.**, "Analise Expermental da Puncao em Lajes de Concreto amadoe Protendido," PhD THESIS, Escodo de Engensharia de Sao Carlos, Universidade de Sao Paulo, Brazil, 2000, 350PP.

22- **Menetrey Ph**. "Synthesis of Punching Failure in Reinforced Concrete". Cement Concrete Compos, 2002; 24: 497-507.

23- **Rezai Kallage, M.,** "Punching Shear Strength of Continuous Post-Tensioned Concrete Flat Plate," MASc thesis, University of Ottawa, Ottawa, Canada, Dec. 1993, 239pp.

24-**Ritz, P., Matt, P., Tellenbach, Ch., Schlub, P., and Aeberhard, H. U.,** "Post-tensined Slabs," Lsinger Ltd.-VSL International, Bern, Sweitzerland, 1981.

25- **Shehata, Ibrahim. A. E, M.,** "Punching of Pre-stressed and Non-Pre-stressed Reinforced Concrete Flat Plate," PhD thesis, Polytechnic of Central London, London, 1982, 226 PP.

26- **Shehata, Ibrahim, A. E. M.,** "Simplified Model for Estimating the Punching Resistance of Reinforced Concrete Slabs," Material and Structures/ Mateiaruxe et Constructions, 23, 1990, PP. 364-371

27- **Shehata, Ibrahim, A. E. M., and Regan, Pual, E.,** "Punching in R.C. Slabs," Journal of structural Engineering, ASCE, V. 115, No. 7, July 1989, PP. 1726-1740.

28- **Silva, R. J. C., Puncao em Lajes Cogumelo Protendida**, PhD thesis, Department of Civil and Environmental Engineering, University of Brasilia, Brasilia, 2005, 223PP.

29- **Silva R.J.C.; Regan, P.E.; and Guilherme, S.S.A.M**., "Post-Tensioned Slabs – Tests and Codes", ACI Structural Journal, V. 104, No. 2, Mar.-Apr.2006, pp. 123-132.

30- **Scordelis, A. C.; Lin, T.Y.; and Mayo, H.R,** Shearing Strength of Pre-stressed Lift Slabs, ACI Journal, V. 55, No. 10, Oct. 1958, PP. 485-506.

31- **Smith, S. W., and Burns, N. H.,** Post-Tensioned Flat Plate to Column Connections Behavior, Journal of Pre-stressed Concrete Institute, V. 19, No. 3, May 1974, PP. 74-91.

32- **Wood, R. H.,** "Plastic and Elastic Design of Slabs and Plates". Thames and Hudson, London, 1996.