Modeling of Transitional Free Hydraulic Jumps

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Abstract Transitional hydraulic jump (T-jump) is formed in expanding channel. The objective of this work is to model the jump sequent depth ratio of such type. Two methods of modeling are achieved. The first is to calibrate a proposed general equation and the second using the artificial neural networks (ANNs). In the second technique, a size of 3-4-1 proposed network provides the best prediction. Three inputs (initial Froude number, toe water depth ratio and the expansion ratio) are utilized. The hidden layer consists of four neurons and used the hyperbolic tangent (tansh) as an activation function. Sensitivity analysis is conducted for inputs. The results of the two methods are compared and the ANN outputs showed better results.

Keywords: Artificial neural network, Open channel hydraulics, Hydraulic jumps, Sudden expansion, Regulators, Stilling basins, Flow modeling.

1 Introduction

Figure 1 shows a definition sketch of the T-jump where a sudden expansion exists in an open channel and a transitional free hydraulic jump is formed. Part of this jump is at the upstream side of the expansion and the rest is at the downstream side, this is called transitional jump (T-jump). This jump can be considered a transition between the perfect jump formed in the narrower channel and spatial jump formed in the wider one.

The T-jump was investigated experimentally by Bremen and Hager (1993, 1994) and Fahmy (2001) and semi-theoretically by Matin et al. (2000). The spatial jump was also examined by Herbrand (1973) and Matin et al. (1997). Other types of jumps as submerged spatial and repelled jumps in expanding channel were analyzed by different authors as Rajaratnam and Subramanya (1968), Smith (1989), Negm et al. (2000), Negm (2000a) and Negm (2002).

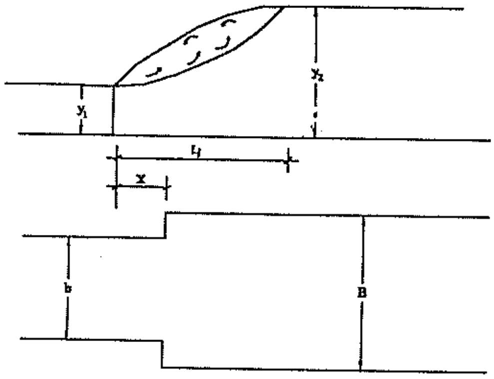

Figure 1. Definition sketch for transitional jump in expanding channel

Bremen and Hager (1993) developed an experimentally based prediction model for the depth ratio Y of the T-jump as follows:

$$Y = Y^{*} - (Y^{*} - 1)(1 - \sqrt{\beta})[1 - \tanh(1.9X)] \qquad (1)$$

in which Y* is the sequent depth ratio of the perfect jump, Y is the sequent depth ratio of the T-jump, $\beta = b/B$ is the expansion ratio where b is the width of the narrow channel and B is the width of wide channel and X is non-dimensional toe parameter which describes the location of the jump at upstream of expansion section and is defined as $X = x/L_r^*$ where x is the distance to the toe of jump measured positively from the expansion section opposite to the flow direction and L_r^* is the length of roller of the classical jump. The length L_r is given by Hager et al. (1990) in the form:

$$\frac{L_r^*}{y_1} = -12 + 160\tanh\left(\frac{F_1}{20}\right), \qquad (2)$$
$$y_1/b < 0.1 \;\&\; F_1 > 2.5$$

2 Proposed Model

The derivation of the depth ratio equation for the classical jumps yields the well known Belanger equation:

$$Y^{*} = \frac{1}{2}(\sqrt{1 + 8F_1^2} - 1) \qquad (3)$$

Equation (3) was developed by applying the 1-D momentum and continuity equations on a control volume of a hydraulic jump assuming that (a) the pressure distribution is assumed as hydrostatic (b) the velocity distribution is uniform (c) effect of turbulence and air entrainment is negligible (d) effect of wall friction is disregarded. Also, the tail-water depth is assumed as the temporal mean value of its fluctuations

Equation (3) can be used for computing depth ratio of different types of jumps, Negm (2000b), when it is written as follows:

$$Y = \frac{1}{2}(\sqrt{1 + 8KF_1^2} - 1) \qquad (4)$$

with K stands for the effect of the various flow, geometrical and boundary parameters of the particular type of jump being under consideration. The factor K is equal 1.0 for the classical jump, Eq.(3). The factor K for the T-jump is function of the initial Froude number, the toe parameter and the expansion ratio, Bremen and Hager (1993). Therefore, one can write the following expression:

$$K = f(F_1, X, \beta) \qquad (5)$$

3 Calibration of the Proposed Model

Calibration of the proposed model given by Eq.(4) involved the determination of the factor K. The experimental results due to Bremen and Hager (1993) and (1994) are utilized for calibration purpose. Several plots were prepared to investigate the nature of correlation between the factor K and the three factors of Eq.(5). Figure 2 presents the typical variations of K with X at different values of F_1=4, 5, 6, 7, 8, 9, and 10, for β=0.5. Also, Figure 3 shows the typical variations of K with F_1 at different values of X=0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 and 1.4, also for β=0.5. These figures show that the relationships among K, F_1 and X may be represented by nonlinear functions. On the other hand, Figure 4 shows the variations of K with β at the same values of F_1 and constant X=0.4. Similarly, Figure 5 presents the variations of K with β for different X at F_1=6. Figures 4 and 5 indicated that the relationships between K and β may be represented by a linear function for fixed X and by nonlinear function for fixed F_1.

The previous figures give a good

support for Eq.(4) to be used for the T-jump modeling. Several functions were proposed and the computed values of K from each function are compared to the experimentally based Eq.(1) due to Bremen and Hager (1993). The following equation was found to provide the best results for K:

$$K = a + bK_1 \tag{6}$$

in which a and b are functions of the expansion ratio β while K_1 is a function of X and F_1.

$$a = 0.9739 + 0.0801\beta - 0.0814\beta^2 + 0.028856\beta^3 \tag{6a}$$

$$b = -0.6506 + 0.6592\beta \tag{6b}$$

$$K_1 = (1 - \tanh[1.9X])F_1^{0.175} \tag{6c}$$

Figure 6 shows the comparison between the experimentally based values of K and the computed values of K from Eq.(6). Clearly good agreement was obtained proofing that Eq.(6) represents the variations of K with F_1, X and β.

4 Verification of the Proposed Model

Figure 7 shows the comparison between the experimentally based values of Y from Eq.(1) and those predicted using the proposed model given by Eq.(4) with K defined by Eq.(6). The figure shows that the model simulated the values of Y very well and could be used in the computations of the depth ratio of the T-jump.

5 Sensitivity Analysis

Equation (4) is used to compute the depth ratio of T-jump, Y for different values of X, F_1 and β. Typical relationship between Y and X for different F_1 at β=0.5 is shown in Figure 8. It is clear that Y is more sensitive for lower values of X than higher values at the same F_1 while at the same X, Y is slightly, more sensitive at higher F_1 than at lower F_1. Other plots at different values of β indicated that the values of Y are more sensitive at lower values of β than at higher values. Similar plots of Y with F_1 for different X at fixed values of β, Y with β for different F_1 at fixed X and third set to show the relationship between Y and β for different X at fixed values of F_1 are shown in Figures 9, 10 and 11 respectively. Figures 9 and 10 show that Y is very sensitive to changes in F_1. While figure 11 indicated that Y is more sensitive to changes in b/B for small values of X (spatial X=0 and T-jump) and very less sensitive to changes in b/B at very large values of X as the jump tends to be classical jump.

6 Modeling Using Artificial Neural Networks (ANN)

The ANNs are recently developed computational tools that can be used for modeling based on learning from examples. The basics of applying this technique in the field of hydraulics was introduced by Negm (2001c) in the Egyptian Journal for Engineering Science and Technology. Also, the basics of ANNs can be found in any textbook such as Schalkoff (1997). Several applications of the ANN were presented in the Journal of Civil Computing, Proceedings of ASCE during the period 1990-2002. Several Water Engineering applications are shortly reviewed by Negm (2001c). The technique was used to forecast the natural flow of the Nile River by Antar et al. (1997). Examples of applications of the technique in hydraulic engineering were developed by Dibike et al. (1999a,b,c), Negm (2001c) and Negm et al. (2002).

In this research, an ANN model of size 3-4-1 was found suitable to produce accurate generalization of the values of the depth ratio of T-jump. Figure 12 shows the developed network to model the present application. The number of neurons of the hidden layer was determined by solving the application several times. In order to start the learning process or training of the network, the weights of the links between the neurons were initiated by assuming random values in the range of ± 0.01 after performing many computer experiments. Also, the activation function was selected by trials. The hyperbolic tangent function was found to be the most suitable one as the system error was significantly reduced. The data was divided into three sets

(i) training data to train the network, 426 data vectors

(ii) validation data set to measure the performance of the network, 53 data vectors and

(iii) test data set to test the network by generating outputs (Y) from only known inputs (X, b/B and F_1), 53 data vectors.

Figure 13 shows the comparison between experimental results based on Bremen and Hager (1993), Eq.(1) and the prediction of ANN for validation and test data sets. It is clear that a good agreement was obtained showing the merits of using the ANN in modeling the depth ratio of T-jump.

The developed ANN model was used to conduct a sensitivity analysis by running the model several times and in each run one variable is removed. The results of sensitivity were presented in terms of the mean relative error and the correlation coefficient in Figure 14. Clearly, F_1 has the major effect, then b/B and finally X is the less effective.

In order to ensure that the developed ANN model is stable, seven computer experiments were performed by randomly selecting the data sets in each test. It was found that an average correlation coefficient of validation data set is 0.9997 of the seven tests which is representative. The variations are in the fourth decimal only. Also, a mean relative error of 0.0051 was found to be a representative for the mean relative error of the seven tests. The standard deviations of the correlation coefficient and the mean relative error are very small with values of 0.0007 and 0.0001. These results are presented in Figure 15.

7 Conclusions

The sequent depth ratio of the transitional hydraulic jump formed in expanding channels was modeled by two approaches. A simplified model in the form of modified Belanger equation was proposed and calibrated using previous published results of other authors. The model provides good results in addition to its simplicity and compacted form. In the second approach, the recently developed computational ANN tool was used to model the depth ratio. A model of size 3-4-1 provided excellent results compared to other approaches. The developed ANN model was also used to study the relative contribution of each of the input variable on the depth ratio.

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Notations

b	width of channel upstream of sudden expansion,
B	width of channel downstream of sudden expansion,
Y	depth ratio of hydraulic jump,
Y^*	depth ratio for classical jump,
F_1	approaching flow Froude number,
Lr	length or roller of jump with L_r^* for the classical jump,
K	defined parameter,
X	the non-dimensional toe parameter for T-jump,
.x	is the distance upstream the expanding section where the T-jump begins,
y_1	initial depth of jump,
y_2	sequent depth ratio, and
β	expansion ratio, b/B.

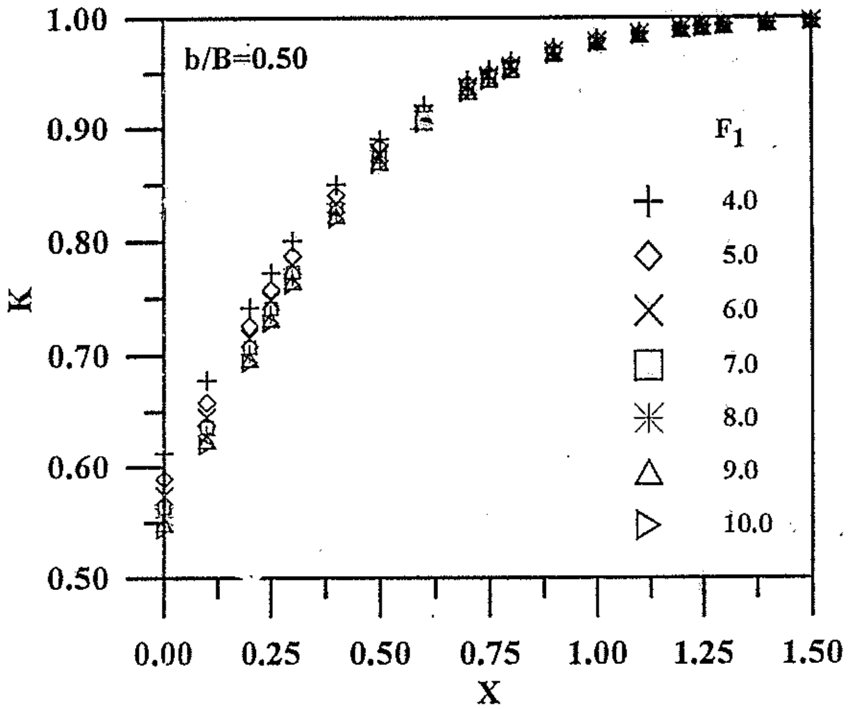

Figure 2. Typical variations of K with X for different F_1

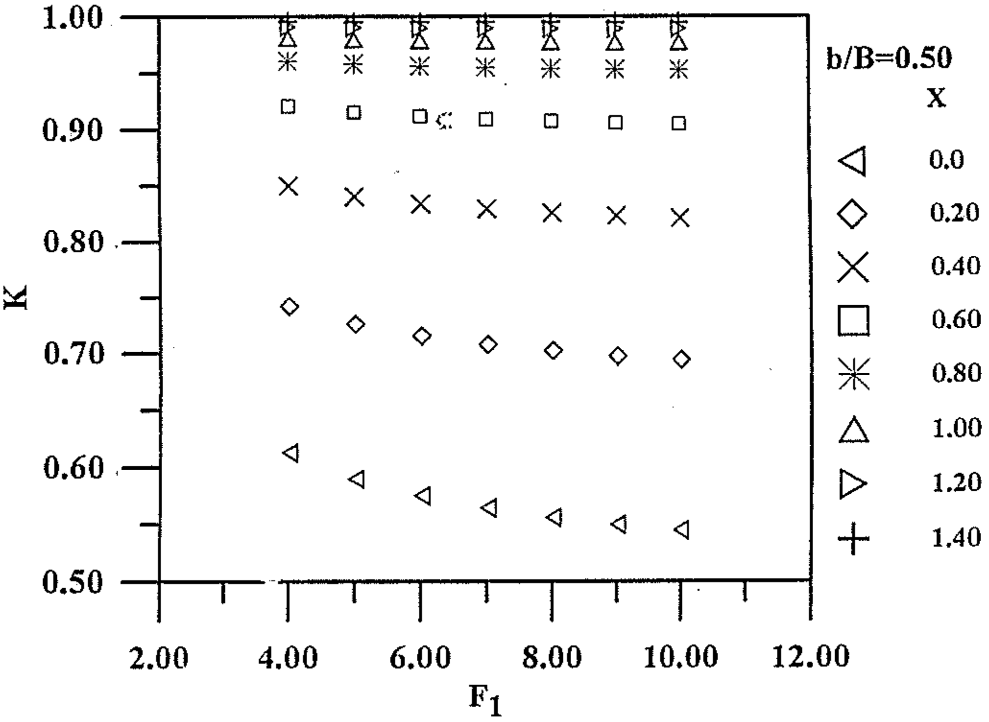

Figure 3. Typical variations of K with F_1 at different X

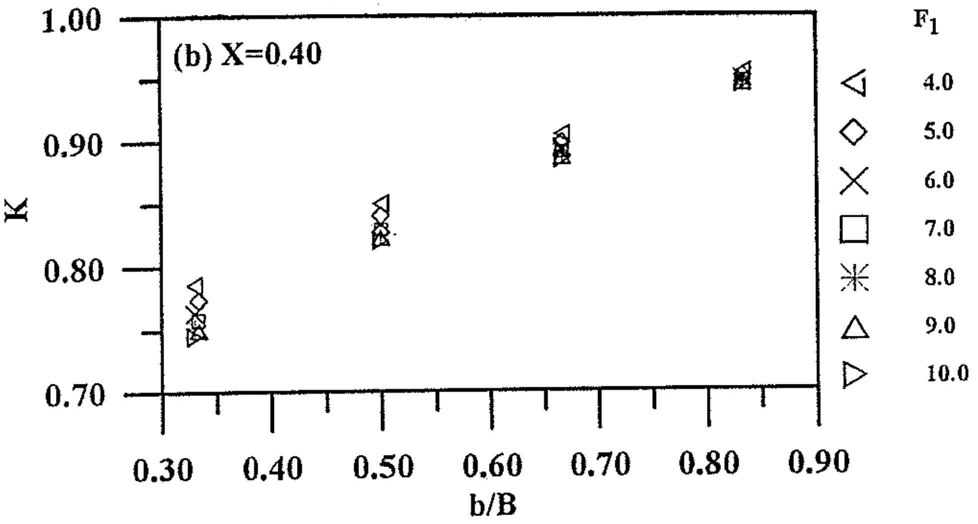

Figure 4. Typical variations of K with b/B for different F_1

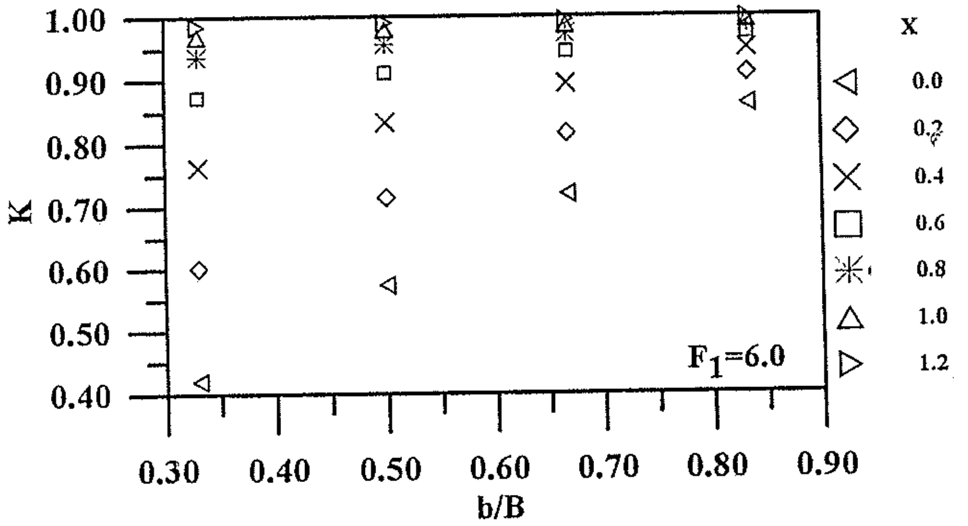

Figure 5. Typical variations of K with b/B for different X

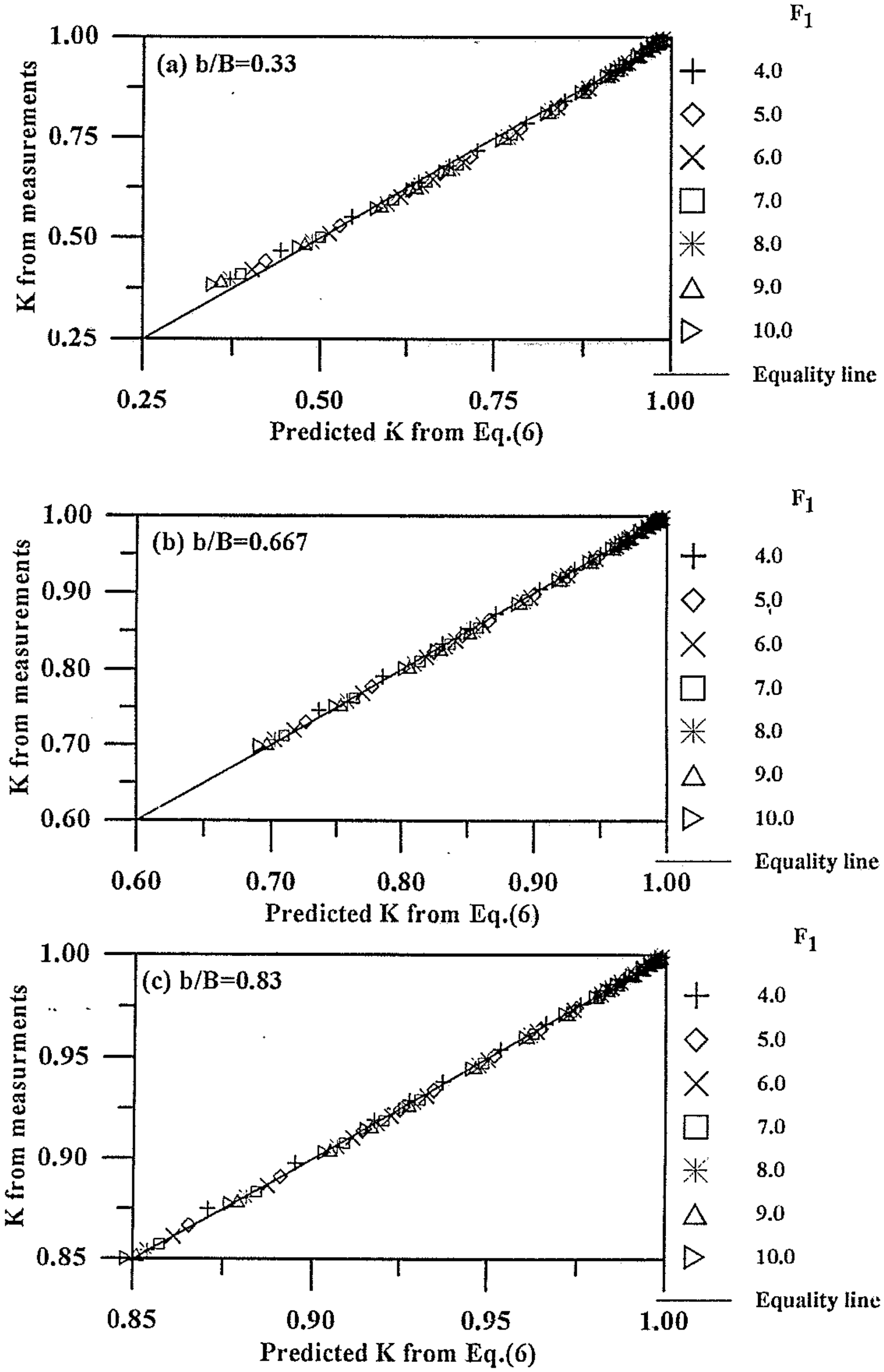

Figure 6. Comparison between predicted K's and those based on measurements for different values of b/B

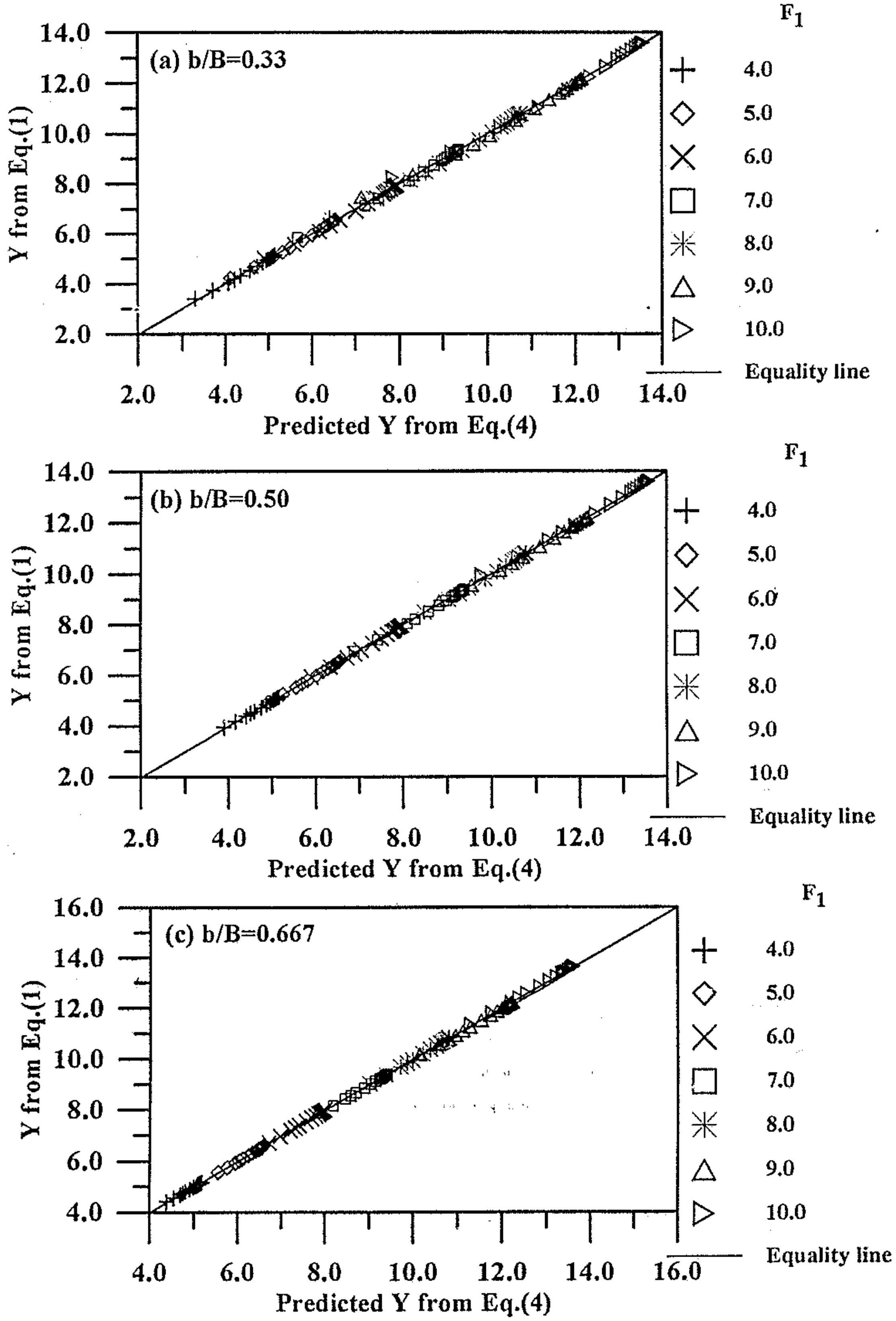

Figure 7. Comparison between predicted Y's and those based on measurements for different values of b/B

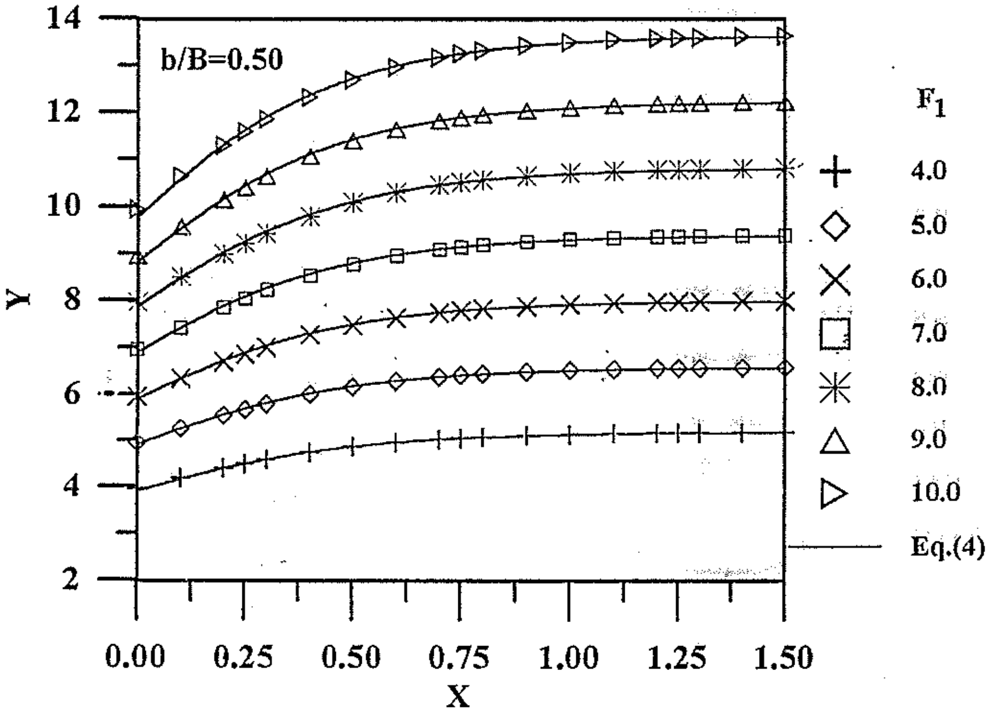

Figure 8. Typical relationship between Y and X for different F_1 according to Eq.(4) compared to generated data from Eq.(1).

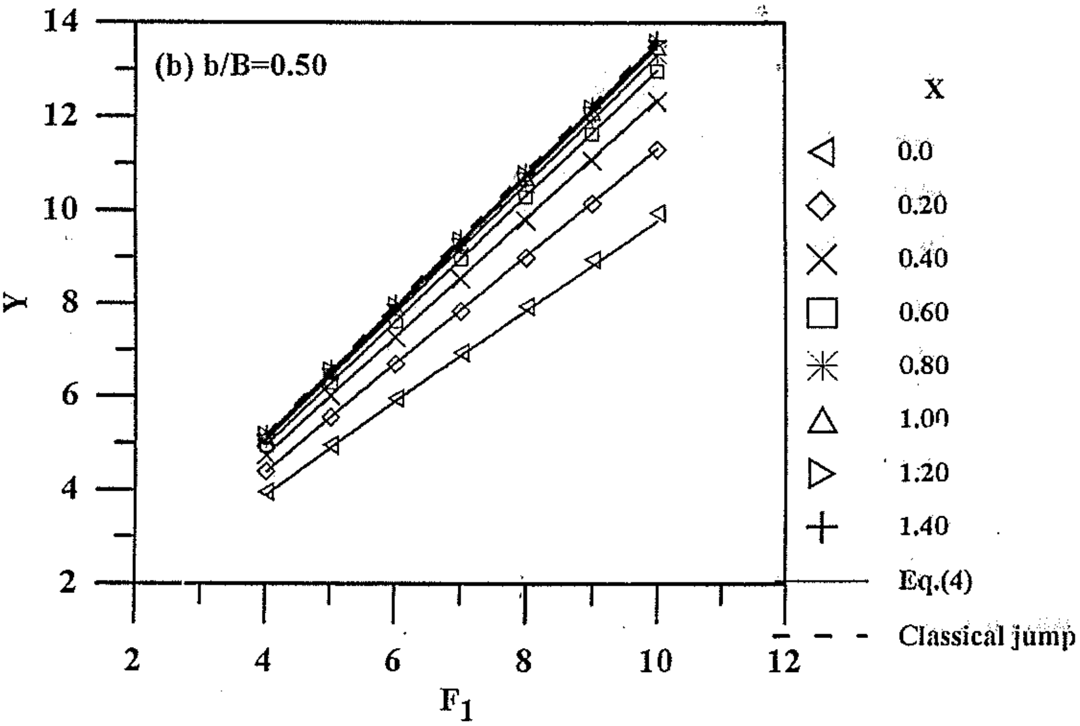

Figure 9. Typical relationship between Y and F_1 for different X according to Eq.(4) compared to generated data from Eq.(1).

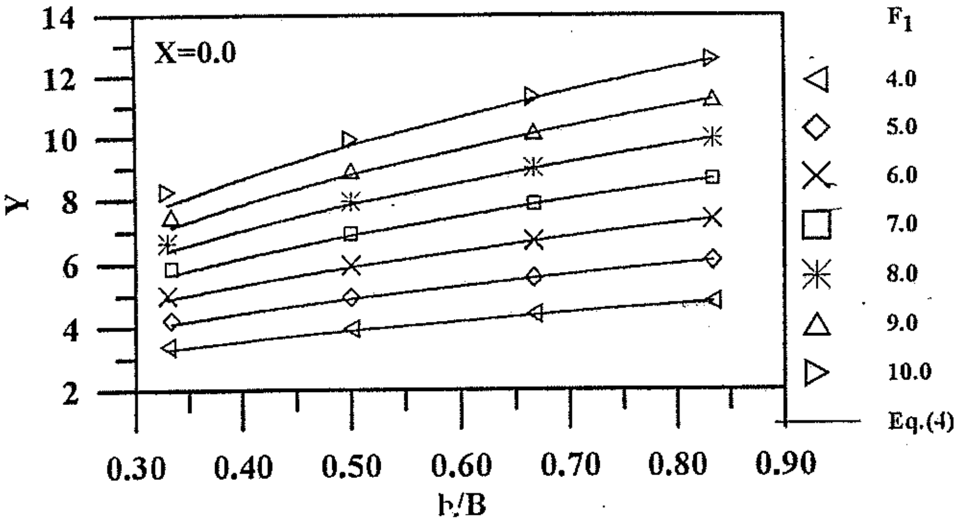

Figure 10. Typical relationship between Y and b/B for different F_1 according to Eq.(4) compared to generated data from Eq.(1).

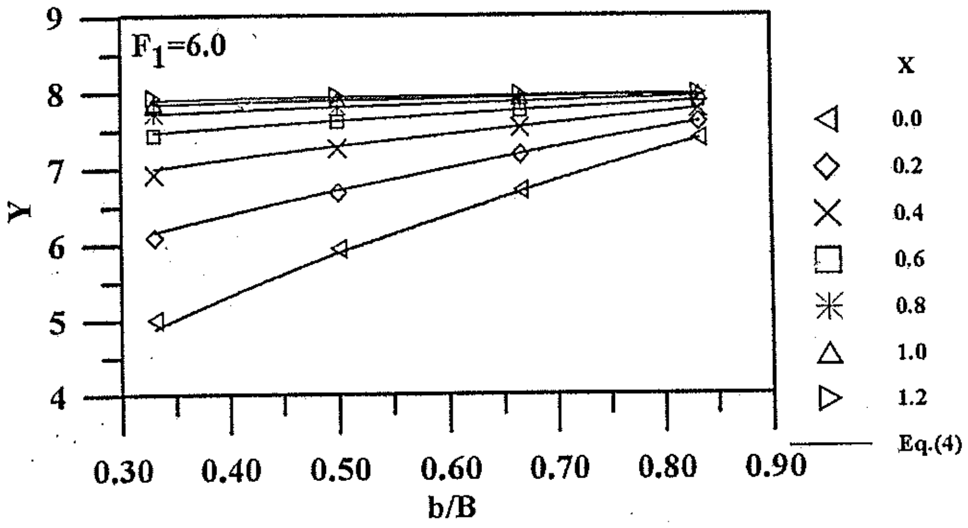

Figure 11. Typical relationship between Y and b/B for different X according to Eq.(4) compared to generated data from Eq.(1).

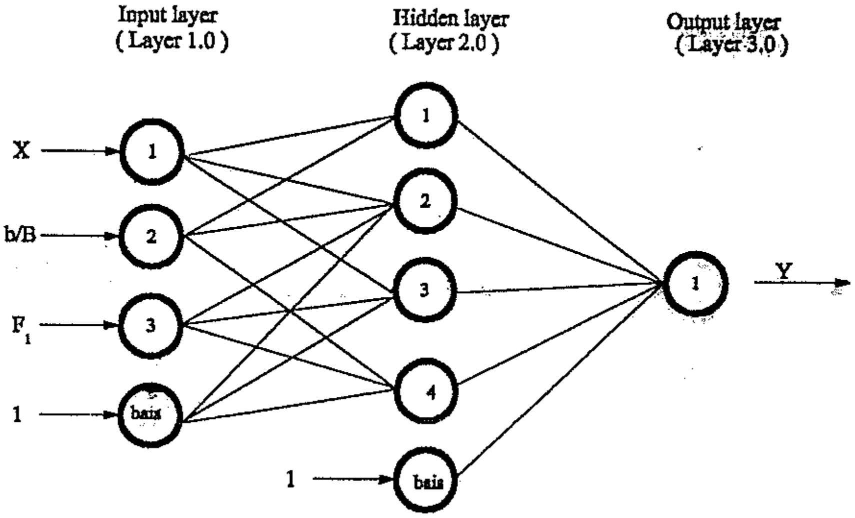

Figure 12. The developed ANN model of size 3-4-1 for the sequent depth ratio of T-jump

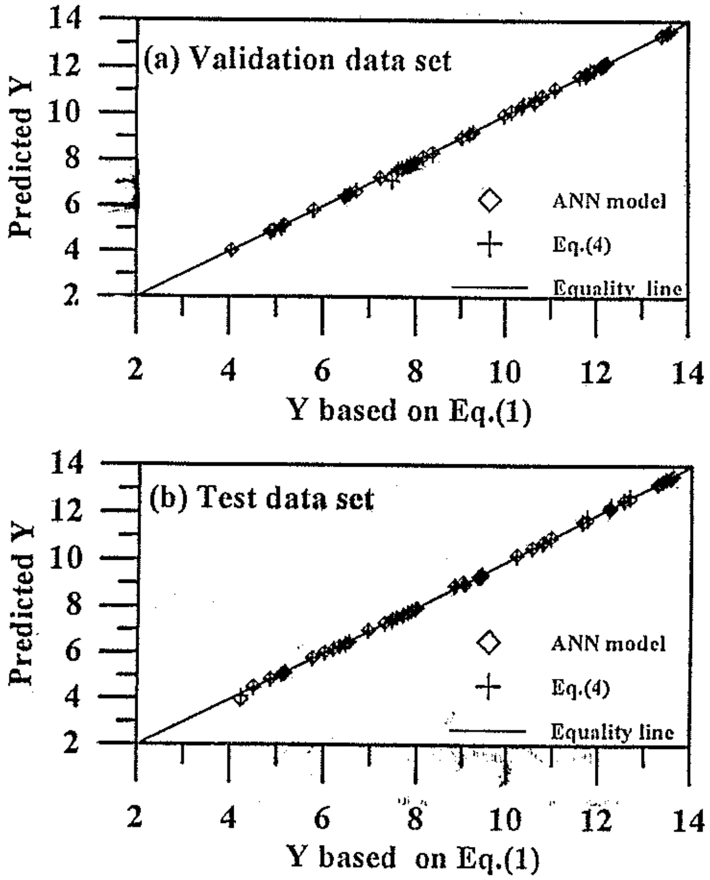

Figure 13. Comparison between ANN prediction and generated data based on Eq.(1) which is based on experimental measurements

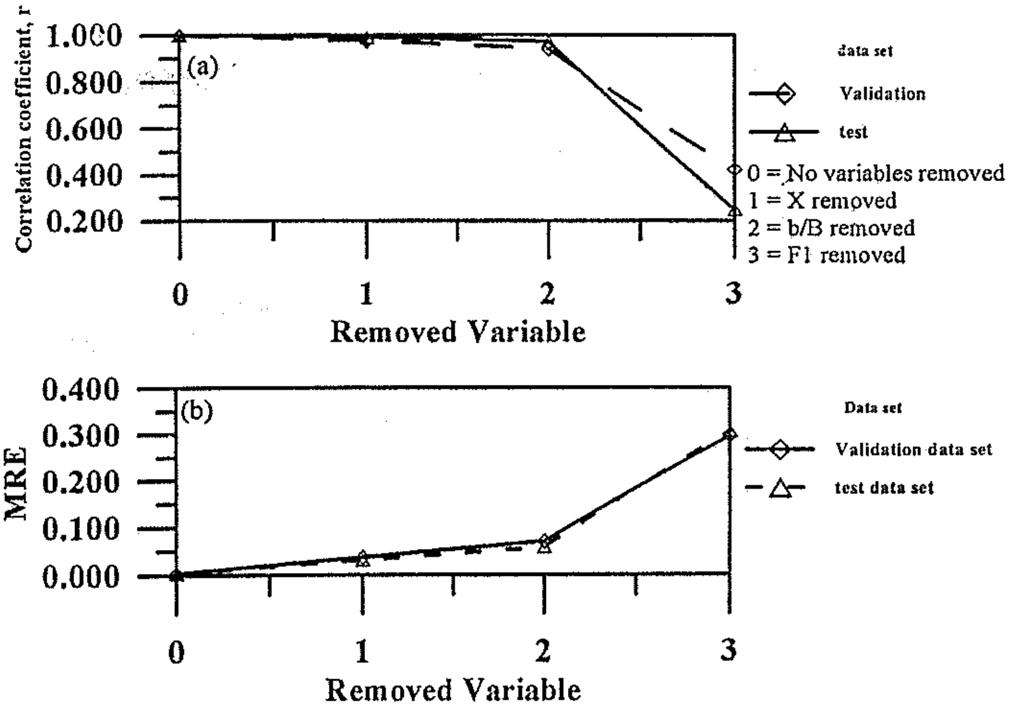

Figure 14. Results of sensitivity analysis in terms of (a) correlation coefficient and (b) mean relative error (MRE)

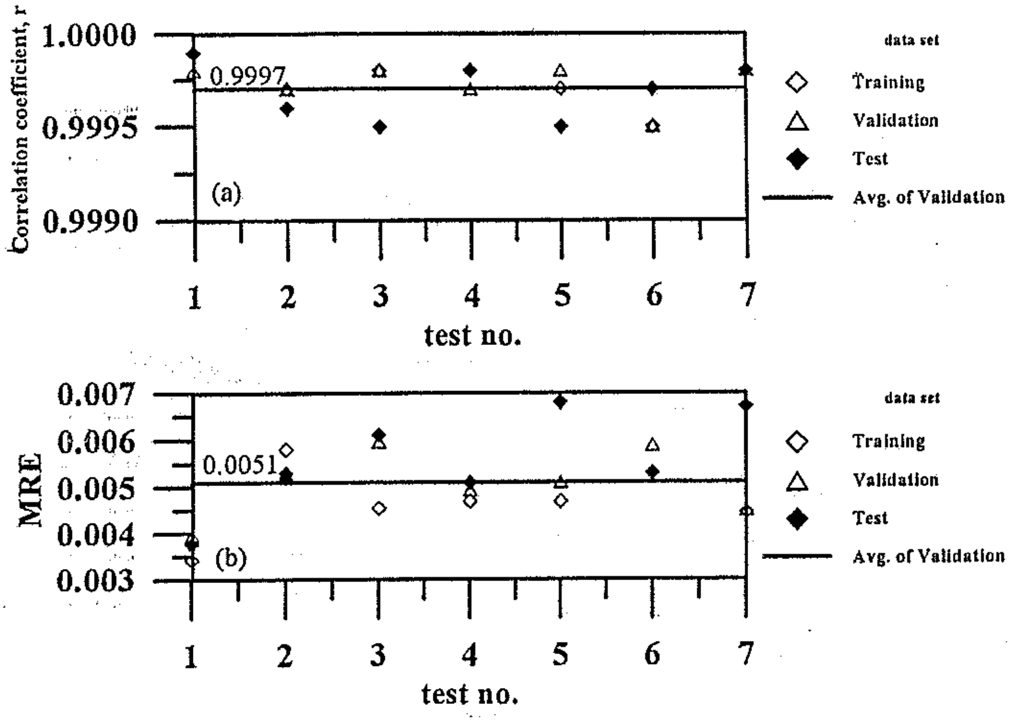

Figure 15. Results of network stability in terms of (a) correlation coefficient and (b) mean relative error (MRE)